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ABSTRACT

Expanded abstracts and critical analyses are given for each of 14 research articles. Four articles involve variables affecting student achievement and attitudes; two each on teaching abstract concepts to young children, problem solving, conservation in young children, and children's understanding of logic and mathematical relations; one on variables affecting simple computation; and one on improvement of attitudes toward mathematics by preservice elementary school teachers. A list is given of all mathematics education research reports included in ERIC references (RIE and CIJE) the previous quarter. (JP)

Vol. 6, No. 3 Summer 1973

INVESTIGATIONS IN MATHEMATICS EDUCATION

Expanded Abstracts and Critical Analyses of Recent Research

Center for Science and Mathematics Education
The Ohio State University
in cooperation with
the ERIC Science, Mathematics and Environmental Education Clearinghouse

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Summer 1973

NOTES . . .

from the Editor

What kind of research does Investigations in Mathematics Education review? If one scans the tables of contents of each issue, does this list represent the "best" available research in mathematics education? the worst? the most important?

These questions arise from time to time, and are occurring now with enough frequency, that a general word of caution is in order. We select for review only those research reports that have been published in prominent journals, or that have been announced and abstracted in Research in Education. In meeting this publication criteria some of the reports have passed stringent review criteria from the journal's referees. Others, however, may have met only token editorial review before being published. The quality control varies from report to report just as it varies from source to source.

Occasionally we receive suggestions that we should review only the "best" research reports, or that we should not have included a review of a certain report because of major weaknesses that it contained. While we certainly have no intent to glorify "bad" research, we are still optimistic enough to believe that people can benefit from a careful study of past mistakes. Furthermore, we believe that the profession has some obligation to identify and comment on published research which should be cautiously interpreted because of methological weaknesses. Investigations in Mathematics Education is one means of making this comment.

Jon L. Higgins
Editor

MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN RESEARCH IN EDUCATION
April - June 1973

ED 070 019 Learning Games and Student Teams: Their Effects on Classroom Processes. 39p. MF and HC available from EDRS.

ED 070 026 The Relationship of Sex of Teacher and Father Presence-Absence to Academic Achievement. 125p. MF and HC available from EDRS.

ED 070 027 Investigation of the Effects of Allocation of Instructional Time on Pupil Achievement and Retention. 133p. MF and HC available from EDRS.

ED 070 284 Effects of Anxiety Type and Item Difficulty Sequencing on Mathematics Aptitude Test Performance. Tech Memo Number 46. 45p. MF and HC available from EDRS.

ED 070 295 Variability in the Proof Behavior of College Students in a CAI Course in Logic as a Function of Problem Characteristics. Psychology and Education Series. 182p. MF and HC available from EDRS.

ED 070 363 The Application of Short-Term Video-Tape Therapy for the Treatment of Test Anxiety of College Students. Final Report. 141p. Not available from EDRS. Available from Rocky Mt. Behavioral Sciences Institute, P.O. Box 1066, Ft. Collins, Colorado.

ED 070 525 The Effect of the DISTAR Instructional System on First and Second Grade Achievement: An Evaluation of the 1971-1972 Title I Program of Winthrop, Massachusetts. 19p. MF and HC available from EDRS.

ED 070 529 Kindergarten Evaluation Study: Full-Day Alternate Day Programs. 38p. MF and HC available from EDRS.

ED 070 657 A Study of Three Concepts of Probability Possessed by Children in the Fourth, Fifth, Sixth and Seventh Grades. 281p. MF and HC available from EDRS.

ED 070 659 Measuring Mathematics Concept Attainment: Boys and Girls. 34p. MF and HC available from EDRS.

ED 070 660 An Analysis of Content and Task Dimensions of Mathematics Items Designed to Measure Level of Concept Attainment. 42p. MF and HC available from EDRS.

ED 070 662 The Performance of First Grade Students on a Nonstandard Set of Measurement Tasks. 25p. MF and HC available from EDRS.

ED 070 663 The Relative Effectiveness of Two Different Instructional Sequences Designed to Teach the Addition and Subtraction Algorithms. 54p. MF and HC available from EDRS.

ED 070 678 Predicting the Relative Difficulty of Problem-Solving Exercises in Arithmetic. Final Report. 65p. MF and HC available from EDRS.

ED 070 679 Effects of the Analytic-Global and Reflectivity-Impulsivity Cognitive Styles on the Acquisition of Geometry Concepts Presented Through Emphasis or no Emphasis and Discovery or Expository Lessons. 155p. MF and HC available from EDRS.

ED 071 056 The Cloze Procedure as a Measure of the Reading Difficulty of Mathematical English Passages. Final Report. 66p. MF and HC available from EDRS.

ED 071 252 Facilitation of Cognitive Development Among Children with Learning Deficits. Final Report. 360p. MF and HC available from EDRS.

ED 071 564 The First Year of Remedial Mathematics Instruction Under Open Admissions. (A Report on the Results of Several Studies of the Remedial Math Program at City College of New York). Report Number 9. 30p. MF and HC available from EDRS.

ED 071 857 The Effects of Provisions for Imagery Through Materials and Drawings on Translating Algebra Word Problems, Grades Seven and Nine. 255p. Not available from EDRS. Available from University Microfilms (72-4976).

ED 071 858 Effectiveness of Mathematics Laboratories for Eighth Graders. 146p. Not available from EDRS. Available from University Microfilms (72-4565).

ED 071 859 The Relationship of Multiple Embodiments of the Regrouping Concept to Children't Performance in Solving Multi-Digit Addition and Subtraction Examples. 151p. Not available from EDRS. Available from University Microfilms (72-6737).

ED 071 860 The Effect of Mathematics Curriculum Matreials on the Perceived Behavior of Urban Junior High School Teachers of Low Achievers. 225p. Not available from EDRS. Available from University Microfilms (72-403).

ED 071 861 Guidelines for Developing a Mathematics Laboratory. 256p. Not available from EDRS. Available from University Microfilms (72-8425).

ED 071 862 Intraclass Grouping of Low Achievers in Mathematics in the Third and Fourth Grades. 100p. Not available from EDRS. Available from University Microfilms (72-11,900)

ED 071 863 A Comparison of Initially Teaching Division Employing the Distributive and Greenwood Algorithm with the Aid of a Manipulative Material. 149p. Not available from EDRS. Available from University Microfilms (72-11,464).

ED 071 864 A Comparison of Two Methods of Instruction in Multiplication and Division for Third-Grade Pupils. 126p. Not available from EDRS. Available from University Microfilms (72-13,636).

ED 071 876 Student Attitudes Toward Geometry. 138p. MF and HC available from EDRS.

ED 071 879 Sixth Grade Mathematics. A Needs Assessment Report. 132p. MF and HC available from EDRS.

ED 071 923 A Technique for Studying the Organization of Mathematics Text Materials. Final Report. 344p. MF and HC available from EDRS.

ED 072 019 A Study of the Influence of a Learning Development Program on the Cognitive Growth and Learning Skills of Elementary Students. 20p. MF and HC available from EDRS.

ED 072 391 Learning Games and Student Teams: Their Effects on Student Attitudes and Achievement. 21p. MF and HC available from EDRS.

ED 072 599 A Survey of Cognition in Handicapped Children. Technical Report No. 197. 77p. MF and HC available from EDRS.

ED 072 615 A Study of the Effectiveness of a Computer When Used as a Teaching and Learning Tool in High School Mathematics. 212p. Not available from EDRS. Available from University Microfilms.

ED 072 937 An Experimental Study of Relationships Between Mastery of a Superordinate Mathematical Task and Prior Experience with a Special Case. 232p. Not available from EDRS. Available from University Microfilms (72-15,073).

ED 072 938 Attentional and Cardinal-Ordinal Factors in the Conservation of Number. 71p. Not available from EDRS. Available from University Microfilms (72-14,413).

ED 072 939 An Instructional System for the Low-Achiever in Mathematics: A Formative Study. 212p. Not available from EDRS. Available from University Microfilms (72-13,978).

ED 072 940 An Exploratory Study of the Diagnostic Teaching of Heuristic Problem Solving Strategies in Calculus. 548p. Not available from EDRS. Available from University Microfilms (72-15,368).

ED 072 941 An Evaluation of Computer-Assisted Instruction Using a Drill-and-Practice Program in Mathematics. 135p. Not available from EDRS. Available from University Microfilms (72-18,627).

ED 072 979 Negative Instances and the Acquisition of the Mathematical Concepts of Commutativity and Associativity. Final Report. 98p. MF and HC available from EDRS.

MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN JOURNALS AS INDEXED BY CURRENT INDEX TO JOURNALS IN EDUCATION
April - June 1973

EJ 067 549 The Effect of Geometric Enrichment Exercises on the Attitudes Toward Mathematics of Prospective Elementary Teachers. School Science and Mathematics, v7 n9, pp794-800, Dec 72

EJ 067 680 Effectiveness of University of Illinois Committee on School Mathematics (UICSM) Stretchers and Shrinkers and Motion Geometry Materials in Improving Arithmetic Ability. School Science and Mathematics, v7 n9 pp822-827, Dec 72

EJ 068 474 Individualized Instruction in Problem Solving in Elementary School Mathematics. Journal for Research in Mathematics Education, v4, n1, pp6-19, Jan 73

EJ 068 475 The Effectiveness of Discovery and Expository Methods in the Teaching of Fourth-Grade Mathematics. Journal for Research in Mathematics Education, v4 n1, pp33-44, Jan 73

EJ 068 476 A Formative Development of an Elementary School Unit on Proof. Journal for Research in Mathematics Education, v4 n1, pp57-63, Jan 73

EJ 068 640 Effects on Transfer of Training of Constant Versus Varied Training, Group Size, and Ability Level, in Elementary School Mathematics. Journal for Research in Mathematics Education, v4 n1, pp20-25, Jan 73

EJ 068 641 Retention of Probability Concepts: A Pilot Study into the Effects of Mastery Learning with Sixth-Grade Students Journal for Research in Mathematics Education, v4 n1, pp26-32, Jan 73

EJ 068 642 The Symmetric Property of the Equality Relation and Young Children's Ability to Solve Open Addition and Subtraction Sentences. Journal for Research in Mathematics Education, v4 n1, pp45-56, Jan 73

EJ 068 939 Language Factors in Learning Mathematics (Research Review). Review of Educational Research, v42 n3, pp359-385, Sum 72

EJ 070 050 Television and Radiovision in the Teaching of Modern Mathematics: A Comparative Study. British Journal of Educational Technology, v3 n3, pp236-244, Oct 72

EJ 070 495 Are Children's Attitudes Toward Learning Arithmetic Really Important? (Research Review). School Science and Mathematics, v73 n1, pp9-15, Jan 73

EJ 071 393 Effects of Four Noise Conditions on Arithmetic Performance. Perceptual and Motor Skills, v35 n3, pp928-30, Dec 72

EJ 071 566 Effects of Music Used to Mask Noise in Learning Disability Classes. Journal of Learning Disabilities, v5 n9, pp533-7, Nov 72

EJ 071 812 The Effects of a Laboratory on Achievement in College Freshman Mathematics. Two-Year College Mathematics Journal, v4 n1, pp55-59, W 73

EJ 072 342 An Investigation into Relative Performances on a Bilingual Test Paper in Mechanical Mathematics. Educational Research, v15 n1, pp63-71, Nov 72

EJ 059 211 270 SE 506 078

TIME DEVOTED TO INSTRUCTION AND STUDENT ACHIEVEMENT. Begle, E. G., Educational Studies in Mathematics, v4 n2, pp220-224, Dec 71.

Descriptors--*Elementary School Mathematics, *Instruction, *Low Ability Students, *Research, *Time Factors (Learning), Grade 4, Mathematics Education, Number Concepts

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James K. Bidwell, Central Michigan University

1. Purpose

The study investigated the effects of mathematical ability and amount of instruction time on the amount of learning by elementary school children.

2. Rationale

J. B. Carroll proposed (in 1963) that students can achieve to any degree of proficiency provided sufficient quality instruction and time are provided. Further, students vary as to the amount of time required. The SMSG staff conducted a study in which below average students were given two years to complete the same amount of algebra that above average students covered in one year. The below average students did as well as or better than the above average (depending on grade level). If these studies are valid, then all students could have the mastery of mathematics by "adjusting the amount of instruction to the ability of the student and expecting the less able student to take longer to achieve mastery."

3. Research Design and Procedure

Fourth grade classes were taught the same content dealing with base five numeration. The students had had no previous experience with number bases other than ten. An original lesson plan was taught to a pilot class.

Based on the pilot study three lesson plans were devised covering the same content. The first covered one class period. the second two class periods, and the third covered three periods. The plans called for increasing amounts of teaching techniques, review of related material, and practice time. No new concepts were employed in the longer plans.

A 15-item scale, Necessary Arithmetic Operation, was used as a pretest of reasoning ability. It was administered the day before each son plan began. The day after the teaching, a 24-item scale ttest, covering the base five numeration content, was administered.

Three experienced teachers, not the regular teachers, taught each of the lesson plans in four participating schools. The plans were taught in the sequence: two day, one day, three day. A total of 225 students were used in the statistical analysis.

The students were divided into three groups based on the pre-test score: low (L) - scores 9 or less, middle (M) - scores 10, 11, 12, and high (H) - scores 13, 14, 15. Table 1 shows pretest results in the 3 x 3 cells of ability versus treatment.

TABLE 1

Reasoning test mean scores by ability group and length of treatment

Ability group	Length of Treatment		
	1 day	2 days	3 days
L	N = 34 mean = 6.59 σ = 1.71	N = 31 mean = 7.16 σ = 1.53	N = 19 mean = 6.58 σ = 2.04
M	N = 24 mean = 10.87 σ = 0.74	N = 24 mean = 10.96 σ = 0.86	N = 26 mean = 10.96 σ = 0.82
H	N = 14 mean = 14.36 σ = 0.84	N = 28 mean = 13.82 σ = 0.72	N = 25 mean = 13.64 σ = 0.76

4. Findings

Table 2 shows the posttest results. The cell means on the three diagonals running lower left to upper right are approximately equal. A two-tailed *t* test showed that none of the differences on the diagonals were significant at the .05 level. Secondly, the 3 day-L mean is higher than the 1 day-M mean (but not significantly) and the 3 day-M mean is higher than the 1 day-H mean (significant at the .01 level).

TABLE 2

Base five mean scores by ability group and length of treatment

Ability group	Length of Treatment		
	1 day	2 days	3 days
L	mean = 4.97 σ = 2.42	mean = 5.74 σ = 2.34	mean = 6.21 σ = 3.39
M	mean = 6.04 σ = 3.34	mean = 5.92 σ = 2.02	mean = 9.50 σ = 3.09
H	mean = 6.36 σ = 4.03	mean = 8.61 σ = 3.70	mean = 11.08 σ = 2.93

5. Interpretation

The results of the study are in accord with Carroll's Thesis. "Less able students learn as much as more able students if they are provided with enough extra instruction." The study is too small and limited to be definitive. It does constitute a strong argument for further studies along the same general lines.

Abstractor's Notes

No details of the actual content or instructional techniques are offered in the brief report. However, note that the mean score for the 3 day-H group was only 11.08 of a possible score of about 20 (a few transfer items were not included in the data). This suggests a high level of difficulty on the posttest. It is possible that the results would have been less significant if the test were less powerful.

There appears to be nothing dramatic in the study. The results were predictable and would be the same in most similar studies. It is usual to assume that additional time for practice and alternative instructional technique improve skill. Additional related material should also improve understanding. These results should show up strongly in any short term study. We need more definitive studies of a long term study covering a year or two of elementary curriculum.

The current thrust at all levels towards individual progress clearly accepts the thesis of this study as valid. Learning has always been individual and we are currently accepting that thesis as a basis for instruction.

James K. Bidwell
Central Michigan University

ED 062 194 SE 013 686

SPATIAL AND MODALITY EFFECTS IN SIMPLE MATHEMATICAL COMPUTATION.

Deichmann, John and Beattie, Ian, Pub. Date April 1972, Note--10p.; Paper presented at the meeting of the American Educational Research Association, April 1972.
EDRS Price MF-$0.65 HC-$3.29
Descriptors--Addition, *Algorithms, Arithmetic, Basic Skills, *Elementary School Mathematics, Multiplication, *Oral Communication, *Research, Subtraction. *Visual Stimuli

Expanded Abstract Prepared Especially for I.M.E. by Max Jerman, The Pennsylvania State University.

1. Purpose

The objective of the study was "obtain data on the preceptual processes and habits which may enter into simple mathematical computations," and the purpose was "to demonstrate possible performance differences related to the method of presentation of simple mathematical computations."

2. Rationale

It appears that researchers have largely ignored the effects of problem format on the performance of simple computations in mathematics. There appears to be no empirical basis for; (1) problem format (vertical vs. horizontal); (2) the location of the operation symbol within the problem statement; (3) the alternate placement of numerals of one and two digits; and (4) mode of representation effects (visual vs. oral). It is not known whether any of the above factors differentially effect initial learning, retention, or computational ease. Any possible interactions between age or grade-level differences are unknown.

3. Research Design and Procedure

A 3x3x3x2 factorial was used, (V, H, A) x (+, -, x) x (2 digit first, 1 digit first). The scores for each cell was derived from the number of correct responses for the 24 Ss on each of the 10 problems for that cell.

Seventy-two undergraduate education majors were randomly assigned to one of 3 treatment groups.

Group I:	Problems presented in vertical format (V)
Group II:	Problems presented in horizontal format (H)
Group III:	Problems presented in aurally (A)

All groups were given the same set of 180 problems, in a single randomized order. 60 of the problems contained the operation sign +, -, or x which was given on the left for H, top for V, and first for A. For 60 of the problems the symbol was given in the middle and for the remaining 60 problems the symbols were given on the right for H, on the bottom for V, and last for A. Of each set of 60 problems, 20 were addition, 20 were subtraction, and 20 were multiplication. For each twenty, 10 problems presented a two-digit numeral first followed by a one-digit numeral. The 10 remaining problems presented the one-digit numeral first. Each treatment group received the problem set in the same randomized order. The visual presentation, H and V, were made by a slide projector at a 3-second rate with 1-second intervals and the aural presentation was made via tape recorder. Students responded on a numbered form supplied.

4. Findings

The authors reported that there were significant ($p < .01$) results for all four main effects and a significant mode x sign interaction ($p < .01$).

The t scores for the means indicated significant differences in favor of A vs. V ($p <.01$); A vs. H ($p <.001$); and V vs. H ($p <.001$). There were no significant differences in + vs. -, but there were for + vs. x ($p <.001$) and - vs. x ($p <.001$). The summary statistics are presented in Table 1.

Table 1

Summary Statistics

	A	V	H	L	M	R	+	-	x
$\bar{X}$	18.18	16.84	11.64	14.84	16.51	15.34	18.12	17.65	10.84
SD	5.02	4.64	6.09	6.01	5.44	6.34	4.40	4.62	5.84

An analysis of the errors students made was performed using multiple t tests. It was found that horizontal multiplication was most difficult as indicated by the number of omitted answers. Addition was also judged most difficult when the sign did not occur in the middle. "Little difference between presentation modes were demonstrated for incorrect answers." It was concluded that operational errors, performing the wrong operation, have a greater probability of occurrence in the visual mode of presentation.

5. Interpretations

Contrary to their predictions, the authors found the order of difficulty to be A > V > H rather than H > V > A. The other predictions were confirmed and correspond to the results shown in Table 1. Group A could have had the advantage over the two visual presentation groups in that they could have been looking at their response sheets during the presentation. Further, the members of the A group were free to rearrange their input into a form suitable to them individually.

Although dealing with college students rather than children, significant differences were demonstrated for all manipulations of format. The causal factors are unknown. If the aural "minus" was replaced with "subtract" differential results might be obtained.

Abstractor's Notes

1. One wonders what the effect of varying the amount of time allowed for presentation would have had on the reported results. Given enough time, there might have been n.s.d.

2. Did the use of multiple t tests infringe on independence of the data?

3. Why was a single random order of exercises used for all groups? all students? Perhaps there was some interaction between the order of exercises and error rate on the single set of exercises.

4. What was the reliability of the test?

5. Was history a factor? This study seems to have taken only 12 minutes for students to complete (180 prob. x 4 sec. ea.).

Max Jerman
The Pennsylvania State University

ED 066 315 24 SE 014 466
A STUDY OF THE EFFECT OF MATHEMATICS LABORATORIES ON THE MATHEMATICAL ACHIEVEMENT AND ATTITUDE OF ELEMENTARY SCHOOL STUDENTS. FINAL REPORT. Hollis, Loye Y., National Center for Educational Research and Development (DHEW/OE,), Washington, D.C., Pub. Date, Jul 1972, Note--24p.
EDRS Price MF-$0.65 HC-$3.29

Descriptors--*Academic Achievement, *Activity Learning, *Attitudes, *Elementary School Mathematics, High Achievers, Intermediate Grades, Laboratories, Low Achievers, Manipulative Materials, *Research

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Gerald R. Rising, State University of New York at Buffalo.

1. Purpose

"This study investigated the effects on student achievement of the combined use of a nonsimulation game and student teams in mathematics."

2. Rationale

The authors cite minimal empirical support for (1) the use of non-simulation games, and (2) competition among student teams, despite rather widespread advocacy and use of these techniques.

3. Research Design and Procedure

Ninety-six seventh-grade mathematics students in two "low math ability" and two "average math ability" classes taught by the same first year teacher comprised the sample for this nine week study. One average and one low class were assigned to the games-teams (experimental) treatment, the others to the traditional (control) treatment. Pretest and post-test dependent variable measures were administered by a research assistant.

The design was 2x2x2 repeated measures ANOVA with factors: (a) Treatment, (b) Math Ability, and (c) Time. Campbell and Stanley nonequivalent control group design was adopted: increments between pre- and post-tests inferring differential treatment effects. A complementary ANOVA technique suggested by Cronbach and Furby was used to give more information about individuals within the classes.

Dependent variables included: (1) Stanford Achievement Test in Mathematics computations subtest, (2) 25 items from this test related to the topics taught during the study, and (3) a divergent solution test designed by the experimenters (and closely related to the game utilized in the experimental classes.)

The single game utilized throughout the study was **Equations**, first developed by Layman Allen.

4. Findings

a. Experimental treatment positive increments were significantly greater on all three measures: (1) $p<.04$, (2) $p<.03$, (3) $p<.04$.

b. Differential learning occured for the two levels of ability. The low ability experimental class produced the largest gains of all four groups. A significant treatment-by-ability-by-time interaction ($p<.05$) was noted for the divergent solutions test.

c. Regression lines were similar for experimental classes, but contrastingly dissimilar for control classes.

5. Interpretations

a. "Our general conclusion is that combining the nonsimulation game Equations with team competition significantly increased students' mathematics achievement over that of a traditionally taught class. The effect was observed for skills specific to the game as well as more general arithmetic skills."

b. "The games tended to reduce the differential learning rates evident in classes of different ability levels." See 4c.

c. The classroom teacher noted experimental class students "turned on" and working and involved in the team competition. She found it easier to provide individual assistance in the game setting.

Abstractor's Notes

The study does achieve its purpose, the provision of empirical data to support a nonsimulation game played by student teams. The authors provide evidence that they are aware of design problems; wherever possible they have responded to them. (I was sorry that both E classes were morning, both C afternoon.)

The divergent solutions test, as was noted, was content-specific to the E classes. The positive C increment over time suggests communication, hardly a startling fact. That this same communication does not erode the other increments lends further support to the "pedagogical transfer" or "general motivational effect" achieved here.

Gerald R. Rising
State University of New York
Buffalo

EJ 067 211 450 SE 507 035

IMPROVING THE MATHEMATICS ATTITUDES OF PROSPECTIVE ELEMENTARY SCHOOL TEACHERS. Hunkler, Richard; Quast, W.G., School Science and Mathematics, v.72 n8, pp. 709-714, Nov. 1972.

Descriptors--*Attitudes, *Mathematics Education, *Preservice Education, *Research, *Teacher Attitudes, Elementary School Mathematics, Methods Courses, Teacher Education, [Research Reports, Shatkin-Dohner Mathematics Attitude Scale]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Loye Y. Hollis, University of Houston.

1. Purpose

The purposes of this study were:

(a) To determine if a content-method mathematics course designed for elementary education majors improves the mathematics attitudes of prospective elementary school teachers.
(b) To determine if the mathematics attitudes of those prospective elementary school teachers who completed the course were significantly different from those prospective school teachers who had not completed the course.

2. Rationale

Other studies have found a significant positive change in student attitude resulting from completing a course or courses in a mathematics preparatory program for elementary school teachers. However, these studies did not compare students who had completed the course and those who had not completed it. This study makes those comparisons.

3. Research Design and Procedure

The course used in the study was a three semester hour method-content mathematics course taught in the Elementary Education Department. The textbook used was Discovering Meanings in Elementary School Mathematics (Fifth Edition) by F.E. Grossnickle, L.J. Brueckner, and J. Reckzeh. The instructors had a good knowledge of mathematics and were asked to:

(a) display a strong interest in the subject
(b) indicate a desire to have students understand the material
(c) display a good control of the class without being overly strict.

The Shatkin-Dohner Mathematics Attitude Scale was used in the study. This scale consists of twenty-two statements, twelve favorable and ten unfavorable, about mathematics. These statements were scored so as to yield a higher score for the more positive attitude.

There were three populations of sophomore and junior level elementary majors available for the study:

(a) students (approximately 50) who had completed no courses in College mathematics.
(b) students (approximately 100) who had completed a content course in College mathematics.
(c) students (approximately 100) who had completed a content course and who were currently enrolled in the content-methods course.

A random sample of 25 was selected from population (a); 50 from (b); and 50 from (c). No students in either (a) or (b) were enrolled in a mathematics course.

During the first week of the semester the selected students were given the Shatkin-Dohner Mathematics Attitude Scale. Then, during the last week of the semester the Shatkin-Dohner Scale was administered to the same population again.

For all three groups the t-test for correlated samples was used to determine if there was any significant difference between the initial and final mean scores on the Shatkin-Dohner Mathematics Attitude Scale. The analysis of variance was used to determine if there were any significant differences among the final mean attitudinal scores of the three groups when they were adjusted for mean differences in the initial attitudinal scores. All tests for significant differences were made at the .05 level of confidence.

4. Findings

Table I shows the means and standard deviations for the initial and final attitudinal scores of the three groups tested.

Table I. Means and Standard Deviations of Attitudinal Scores.

Groups	Initial Test			Final Test		
	N	M	S.D.	M	S.D.	t
Group 1	25	54.56	16.10	54.12	18.46	-.338
Group 2	50	62.76	22.93	63.70	22.69	.773
Group 3	50	61.32	22.19	66.22	21.02	3.278

Table II shows the comparison between the mean adjusted scores of the groups.

Table II. Individual Comparisons for Adjusted Mean Scores.

Comparisons	d.f.	F
Group 3 vs. Group 1	1 and 121	6.78
Group 3 vs. Group 2	1 and 121	3.89
Group 2 vs. Group 1	1 and 121	1.00

(The numbering of these tables does not correspond to the numbering in the original article.)

5. Interpretations

It was concluded that:

(a) The method-content mathematics course designed for elementary school teachers did improve the mathematics attitudes of the prospective teachers completing the course.

(b) The method-content courses in combination with the content course can probably be used to improve the attitude of teachers who have had no college mathematics preparation.

Abstractor's Notes

The study presents a strong case for a mathematics course and a method-content mathematics course being able to change attitudes toward mathematics. Table I raises some interesting questions in that respect.

1. Do the scores made on the initial test not show a strong positive influence being produced by the mathematics course?
2. Since there is a difference in M of 6.76 between Group 1 and Group 3 on the initial test and a 4.9 difference in M between Group 3's initial test score and final score, is there an indication that the mathematics course influences attitudes more than the method-content mathematics course?

It would have been helpful it the researchers had more completely explored these areas. Although the author's Table 2 was presented to present data showing there were no significant differences in the initial attitudinal scores, for me, it falls short and does not resolve the questions.

Loye Y. Hollis
University of Houston

ED 059 039 SE 013 148
INSTRUCTION IN PROBLEM SOLVING AND AN ANALYSIS OF STRUCTURAL VARIABLES THAT CONTRIBUTE TO PROBLEM-SOLVING DIFFICULTY. Jerman, Max, Stanford University, California Institute for Mathematical Studies in Social Science, Pub. Date, Nov 12, 1971, Note--129p.
EDRS Price MF-$0.65 HC-$3.29

Descriptors--*Arithmetic, Computer Assisted Instruction, *Educational Research, *Elementary School Mathematics, Grade 5, *Mathematics Education, *Problem Solving, Programmed Instruction.

Note: The first part of this report contains the major sections of the author's doctoral dissertation, "Problem Solving in Arithmetic as Transfer from a Productive Thinking Program" (Stanford University, 1971), which is also reported in "Individualized Instruction in Problem Solving in Elementary School Mathematics," Journal for Research in Mathematics Education, v4 n1, pp6-19, Jan. 1973.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Jeremy Kilpatrick, Teachers College, Columbia University

1. Purpose

The purpose of the dissertation study was to investigate the relative effects of two self-instructional programs--one teaching general heuristic strategies, the other teaching specific techniques--on fifth graders' ability to solve word problems in arithmetic. The purpose of the additional analysis, discussed in the second part of the report, was to investigate how well certain variables associated with problem structure would predict the difficulty of the word problems.

2. Rationale

Two approaches to teaching problem solving have been advanced recently and have been shown to be successful on their own terms: one emphasizes instruction in general problem-solving skills not oriented to any particular academic discipline; the other is a modification of the traditional "wanted-given" approach for teaching children to solve one-step word problems in arithmetic. The relative effectiveness of programs embodying the approaches, as measured by tests of mathematical problem-solving skills, had not been investigated.

In previous studies, linear regression models were used with some success to predict the difficulty of word problems in terms of variations in the structure of the problems (length in words, number of steps needed for solution, etc.). The earlier data had been gathered during computer-assisted instruction, and the question arose as to how well the same structural variables would predict the difficulty of problems presented in the paper-and-pencil setting of the dissertation study.

3. Research Design and Procedure

One program was The Productive Thinking Program, a commercially-published set of programmed booklets that present, in a comic book format, the continuing story of a brother and sister team who, with the aid of their uncle, try to solve a series of (non-mathematical) puzzles and mysteries. The pupil is supposed to identify with the children as they gradually become more proficient in problem solving. The program is intended to increase existing skills rather than creating new ones. The pupil is given practice and encouragement in such strategies as planning one's attack, searching for uncommon ideas, and transforming the problem.

The other program was Jerman's Modified Wanted-Given Program, which presents, also in a programmed, comic book form, the story of a man and boy as they encounter and solve a series of arithmetic problems. The pupil is taught to classify word problems as either sum problems or product problems. Then he is taught to perform the appropriate operation depending on what is wanted and what is given. Multiple-step problems are solved by an iteration of the basic approach.

The two programs were assigned at random to students in each of six classes; two other classes, which received no instruction in problem solving, served as controls. The 261 subjects were fifth graders in four public schools. Each program consisted of 16 booklets; one booklet was studied each day for 16 consecutive school days.

The pretest contained four scales: a test of inductive reasoning using geometric figures, a test of the ability to perform operations according to written directions, a test of reasoning ability in which the subject must choose which arithmetic operation is needed to solve a word problem, and a test of flexibility of closure in which the subject must find a figure hidden in a complex pattern. The posttest contained three scales: a repetition of the test of the ability to perform operations on whole numbers according to written directions, a test of the ability to read a passage about an unfamiliar mathematical idea and answer questions about it, and a set of five arithmetic word problems. The follow-up test, given seven weeks after the posttest, contained three scales: a test of the ability to determine the missing digits in a computation in which letters replace some of the digits, a second test of the ability to read a passage about an unfamiliar mathematical idea and answer questions about it, and a second set of five arithmetic word problems.

The two programs and the control treatment were taken as treatments in a 3x2 factorial design (with sex as the second factor). The four pretest scales were used as covariates in several analyses of covariance of the scores on the posttest and follow-up test scales. (In an attempt to explore the lack of treatment differences, subsequent analyses of covariance were run with the posttest and follow-up word problem scales rescored according to whether the correct procedure was used, regardless of the solution.)

For analyses presented in the second part of the report, data were obtained on the difficulty of 29 problems solved as part of the dissertation study. Nineteen had been solved by pupils ($n = 20$) as part of the Modified Wanted-Given Program; ten came from the two sets of arithemtic word problems taken by the pupils ($N = 161$) as part of the posttest and follow-up test. Thirteen variables that in earlier CAI studies had accounted for variance in problem difficulty and six newly-defined variables were used in several stepwise regression analyses of data from both CAI and paper-and-pencil sources.

4. Findings

There were no significant differences in criterion test performance or in mean gains on two tests: the test of ability to perform operations on whole numbers according to written directions (posttest - pretest) and the test of arithmetic word problems (follow-up test - posttest). There was only one significant sex difference ($p < .05$, favoring boys on the posttest scale dealing with an unfamiliar mathematical idea) and no significant treatment-by-sex interaction on the criterion tests.

There were significant differences ($p < .001$) among the treatment groups in their use of the correct procedure in solving the word problems on the posttest and the follow-up test. Analyses of covariance with the control group omitted indicated that the Modified Wanted-Given treatment was superior to The Productive Thinking Program in eliciting correct procedures on the posttest ($p < .005$) but not on the follow-up test. A significant treatment-by-sex interaction for use of the correct procedure on the posttest ($p < .05$ for all three groups, $p < .025$ for the two experimental groups; boys superior on the Modified Wanted-Given Program, inferior on the other two) disappeared on the follow-up test.

Data obtained from a previous CAI study were used to suggest and test additional structural variables that might predict problem difficulty. The multiple R for the CAI data rose from .67 to .82, .84, and .85, respectively, as the number of variables was increased from 6 to 16, 19, and 22 (with some redefinition of variables and a drop in the number of problems from 68 to 65).

Eleven of these variables were used to predict the difficulty of the 29 dissertation problems with a multiple R of .77. Two new variables increased R to .80; two additional variables used previously increased it to .83; and four more new variables (and one additional problem) increased it to .95. These final four variables made a substantial difference in the prediction equation: three of them were among the first five (for which $R = .93$) in the analysis using all 19 variables.

When the 19 variables were used with the data from the CAI study, the fit was not as good ($R = .75$ for the first five variables) and the order of entry was quite different from that for the paper-and-pencil data. Length of the problem in words was the only variable among the first five in both analyses.

5. Implications

The results seem to indicate some degree of independence between fifth graders' computational skill and their ability to adopt an appropriate solution strategy. "It would appear that teaching problem solving in mathematics to fifth graders can best be done in a mathematical context using a wanted-given approach. One should not necessarily expect gains in the number of problems correctly solved, unless the teaching of problem-solving strategies is accompanied by instruction in the appropriate computational skills [pp. 83, 84]."

Systematic instruction appears to be more effective than no systematic instruction in helping students use appropriate problem-solving strategies.

Preliminary evidence on the structural variables contributing to word problem difficulty suggests that the tools at the students' disposal may have an important effect. The student using a CAI system does not need the computational facility that the student using pencil and paper must have, and consequently the variables influencing problem difficulty are different in each case. Nonetheless, relatively good prediction of difficulty can be achieved.

Abstractor's Notes

The two parts of the report deal with somewhat disparate issues and ought to have been published separately. The dissertation study is an important addition to the literature on instruction in problem solving, but flaws such as the possible inappropriateness of one of the treatments (the subjects had much more difficulty with the arithmetic computations in the Modified Wanted-Given Program than the author anticipated on the basis of pilot study data) limit the generalizability of the findings. A further limitation is that the significant differences between groups emerged in an a posteriori analysis, after tests of the main hypotheses revealed no significant differences in performance on the various criterion tests. The author gives insufficient attention to this limitation, in my view.

An analysis of covariance of the number of correct procedures used by the subjects yielded a significant difference "in favor of treatment groups 1 and 2." How did the author decide that this was, in fact, the source and nature of the difference? What is meant by a significant treatment-by-sex interaction "in favor of boys "? These questions are especially disturbing in view of the author's otherwise careful approach to instrumentation, design, and analysis.

The prediction-of-problem-difficulty study is much the weaker of the two. It has an ad hoc character ill-suited to a study purporting to validate earlier work. The sequence of regression analyses is difficult to follow, with variables renumbered confusingly several times, new variables created apparently to suit the particular set of problems used, and arbitrary changes in both the set of variables and the set of problems. The approach is entirely atheoretical, and no provision is made for cross-validation of results. This work is important only as it leads to more carefully designed studies, and it deserves publication about as much as an artist's preliminary sketches or a writer's first draft.

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PREDICTING THE RELATIVE DIFFICULTY OF VERBAL ARITHMETIC PROBLEMS. Jerman, Max; Rees, Raymond, Educational Studies in Mathematics, v4 n3, pp306-323, April 1972.

Descriptors--*Arithmetic, *Mathematics Education, *Problem Solving, *Research, Elementary School Mathematics, Matbematics.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Marilyn N. Suydam, The Ohio State University.

1. Purpose

To attempt to identify and define a meaningful set of variables that can account for a significant amount of the variance in the difficulty level of problems solved correctly.

2. Rationale

A basic assumption is that the structure of the arithmetic word problem to a large measure determines its difficulty. Research on variables considered by Suppes and others in a CAI context were discussed. The variables which appeared most robust were: Operations; Length; Division; S_2, the internal sequence variable; and Conversions. Memory and Distractor Cues may or may not play important roles in subsequent analyses. These analyses were performed using a regression model on CAI curriculum where students indicated the operations to be performed, but not actually performing the computations. The present research is to determine if these same variables were applicable to problems solved with pencil and paper.

3. Procedure

Eleven of the 22 variables identified from the previous CAI research were tested on word problems solved off-line to see if their order of entry in the stepwise regression was at all similar to that found in on-line CAI context. These variables were first tested on 29 problems selected from three sets solved correctly using paper and pencil by average fifth-grade students. (Nineteen problems were from one study, n = 20; two sets of five problems had been used as posttest and retention test in another study, n = 116.) Stepwise regression indicated that Length (the number of words in the statement of the problem) entered first, followed by Memory, S_2, S_1, and Verbal Cue. The variance accounted for by nine variables was .595. The variables that accounted for most of the variance using CAI were also effective, though at a lower level in paper and pencil solving.

In an attempt to account for more of the variance, eight new variables were defined. The 29 problems were again subjected to regression analysis, using all 19 variables.

4. Findings

A linear regression model was stated which "gave a surprisingly good account of the difficulty level of a somewhat artificially arranged set of verbal problems for fifth grade students." Five variables (of 19) were found to account for almost 87 percent of the variance in the observed probability correct: the multiplication variable, the division variable, the number of words in the problem statement, the verbal distractor variable, and the addition-subtraction variable.

5. Interpretation

That only five variables accounting for 87 per cent of the variance were identified was considered good, because it is not practical to take into account more than four or five variables when writing word problems. An equation has been derived for predicting the relative difficulty of verbal problems for fifth graders, although it is not simple to implement. A good deal of replication is needed to comfirm or deny these findings.

Abstractor's Notes

In essence, this is a progress report. The research reported is one of a series of studies completed, underway, or planned. The results of this study may be useful to other researchers; they are certainly not of the type that can be implemented or used by curriculum developers -- as is pointed out in the study.

At the time I first read this study, I noted: "The variables need to be tested with students who actually solve an entire set of problems from which the variables and thus the equation were derived. Until the set of variables is definitive, and they can be applied with accuracy in the construction of verbal problems, it is difficult to ascertain the contribution made by these studies on the difficulty level of problems."

Since then there have been several further studies; one of these is presented as an example of "correcting" an observed weakness in previous studies:

> Jerman and Mirman (1972) prepared eight sets of 20 word problems (included in an appendix) of a predicted level of difficulty based on six variables (multiplication, division, recall, conversions, operations, and number of words in the problem statement) identified in the Jerman and Rees study and two follow-up studies. Four problem sets were administered to students in grades 4-6 ($n = 161$) and four different sets were administered to students in grades 7-9 ($n = 179$). The regression equation cited by Jerman and Rees did not yield accurate predictions for

these groups, based on a chi-square test. New equations were computed for each grade level, and the problems were recorded in terms of the six variables which gave the best account of the observed varience in probability correct at each grade level. The new equations gave more accurate though not yet "satisfactory" predictions. However, relative difficulty remained constant at different grade levels. It was concluded that the set of variables needs to be further improved so the predictive power of the regression equation is improved.

To repeat: until the set of variables is definitive and the efficiency of their use is determined, it is difficult to determine the contribution of these studies.

The need for a consistent attack on curriculum problems has been stated often. It is gratifying to observe such an attack, to trace the use of previous results, the refinement of techniques, the "correcting" of weaknesses in previous studies. One wishes that the focus of the research involving so much effort were of more significance . . .

Some further thoughts:

(1) Verbal problems are included in the curriculum largely because they provide a logical (if admittedly artificial) way of helping students learn to cope with real problems met outside the classroom. Are problems in real life to be ordered by difficulty? Should the emphasis be on difficulty level -- or on meaningful content?

(2) At a time when we speak of relevance and begin to define problem solving in a broader sense, it is interesting to see dollars being devoted to the restricted type of problem (on which more research, with undefinitive results, has been done than on any other topic).

(3) It is interesting to conjecture that classroom teachers (who did not know the research results) might very well indicate that problems involving division, regrouping in multiplication, two operations, distractors, and/or "more" words are more difficult for children to solve correctly . . .

(4) Manipulations of data . . . a set of problems is developed according to specifications involving certain variables. Data indicates that the variables are unsatisfactory, so the problems are recoded. How specific are the specifications?

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Reference

Jerman, Max and Mirman, Sanford. Predicting the Relative Difficulty of Problem-Solving Exercises in Arithmetic. Final Report, National Center for Educational Research and Development. December 14, 1972.
ERIC: SE 015 512

ED 061 094 SE 013 556

AN INVESTIGATION IN THE LEARNING OF SELECTED PARTS OF A BOOLEAN ALGEBRA BY YOUNG CHILDREN. Johnson, David C., Pub Date Apr 72, Note--48p, Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, Illinois, April 4, 1972. EDRS Price MF-$0.65 HC-$3.29

Descriptors--Concept Formation, *Elementary School Mathematics, Grade 1, *Instruction, Kindergarten Children, Learning, Logic, Relationship, *Research, *Set Theory

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Mary Ann Byrne, University of Georgia

1. Purpose

To determine if specific instructional conditions improve the ability of young children (grades K-1) to:

a) form classes based on the intersection, union, and negation of attributes of objects.

b) recognize selected equivalence and order relations that exist within and among sets of objects.

To investigate whether transfer from the above learning will occur in:

a) multiplicative classification activities

b) class inclusion activities

c) the use of the transitive property of the selected equivalence and order relations.

To investigate the effects of age and intelligence on the above learning.

2. Rationale

Classification and seriation are at the heart of the theory of Piaget. When asked to classify, children under five simply form figural collections. By age seven, children can sort objects, add classes, and cross-classify classes. However, a genuine operatory classification does not exist until about age 8. An implication of the theory of the Geneva School is that school instruction can not be assimilated adequately without requisite cognitive structure.

The present study represents the employment of this theory in an applied research problem.

3. Research Design and Procedure

A Posttest-Only Control Group Design was used to test 12 major hypotheses. Factors considered were:

a) chronological age (A) with two levels (64-76 months and 77-89 months).

b) IQ as measured by the Otis-Lennon Mental Ability Test with two levels (80-100 and 105-125).

c) treatment (T) with two groups (An experimental group received an instructional unit of 17 lessons and a control group received no instructions.).

The hypotheses were repressed and the statistical analyses done in terms of three 2x2 factorial designs (IQ vs T, IQ vs A, T vs A). Hypotheses concerned the main effects and interaction of the three independent factors on two classes of dependent variables: achievement and transfer.

The 196 children from four kindergarten and four first grade classes within geographic proximity were assigned to the appropriate category determined by their IQ and age. An "ordered random sample" of 80 subjects was then selected; 20 in each of the four categories. Children within the categories were randomly assigned to a treatment group. Another 35 subjects were chosen as alternates.

The 17 instructional lessons were constructed to help children learn to form classes, the intersection, unior, and complements of classes, and relations (more than, fewer than, just as many as) between classes and class elements.

Five basic posttests were constructed to measure the two achievement outcomes and the three transfer outcomes described in the "Purpose" section. However, scoring and statistical analyses were completed using 9 subjects.

Item analyses were made on 99 items using the responses of 115 children (included alternates). Certain items were deleted from further analyses. The 12 hypotheses were tested using univariate and multivariate analyses of variance procedures. The scores of 80 subjects on 72 items were used in these analyses.

4. Findings

Twenty-seven items were excluded after the item analyses because of "undesirable item characteristics." Twenty-six of these excluded items were non-discriminators between experimental and control groups.

The internal consistency reliabilities for the subtests ranged from 0.58 - 0.82. Six of the nine subjects had KR-20 reliability coefficients ≥ 0.70.

MANOVA's using the five achievement tests as response variables and a pair of independent variables showed a significant ($p < .01$) difference in mean vectors for each pair. The experimental treatment group and the high IQ group performed significantly ($p < .01$) better than the control and the low IQ group, respectively, in each analysis. The interaction of T and I was significant at the 0.05 level.

To further interpret the effects of T, I, and TxI, univariate analyses were performed using single subtest scores. The main effects of T and I were significant ($p < .01$) for all subtests in the direction described above. A significant interaction ($p < .05$) occurred on only two of the subtests. On both of these subtests the treatment increased the performance of the high IQ group more than that of the low IQ group.

MANOVA's using the 4 transfer tests showed a significant difference in mean vectors for each pair of independent variables, and a significant main effect for T and I in the directions given above. Univariate analyses showed that main effects were not consistent across subtests.

5. Interpretations

This study presents evidence that kindergarten and first grade children can be taught selected classification and comparing abilities and the increase in achievement is accompanied by some transfer to related activities.

The main effects of treatment and IQ level were very significant on both achievement and transfer measures but the main effect of age was not significant on any measure.

The powerful effect of IQ suggests further study of the relation between Piaget's classification of mental operations and the degree to which these operations are measured on various IQ tests.

Piaget distinguished between physical experience and logical-mathematical experiences. An examination of the subtests and results indicate that the experimental unit produced substantial improvement in physical knowledge but very little improvement in operatory classification.

Abstractor's Notes

It is important to the development of theory that it is employed in empirical studies. It is nice when a theory can aid in providing answers to applied problems in education. The present study sought to employ the theory of the Geneva School in the area of mathematics education.

Here are several reactions and questions regarding the investigation as reported.

1) Most of the pages of this report providing rationale consisted of a review of the eight groupings postulated by Piaget rather than more clarification of how the study was embedded in mathematical theory and directed to a particular mathematics education research problem. The dissertation on which this report is based included such a clarification.

2) Reasons for several procedures were missing. Why were subtests used in the analysis instead of the five posttests as such? Why were three 2x2 analyses used instead of a 2x2x2 analysis? Why were alternates chosen? Why wasn't the Null Hypothesis 7 on interaction of T and I not rejected?

3) There were a few misprints. In Table 2, for subtest CA_3 were 2 nondiscriminators. On page 24, the interaction of T with I was <u>not</u> significant.

4) In general, the methods of scoring were well-explained. However, the explanation of scoring subtest CA_3 could not have been followed given the scores obtained.

5) There was a discrepancy between the discussion of statistics to be used and the results described. The report indicated there would be three MANOVA's with nine response variables. Instead there were six MANOVA's; three with five response variables and three with four response variables.

6) During the study itself 27 items were deleted after the Item Analysis and before the other analyses. It is unfortunate that items were not tested in a pilot situation, rather than being

deleted in the study itself. Twenty-six of the 27 items deleted were non-discriminatcrs. There was no discussion of whether this was taken into account when calculating the test statistic for treatment groups and when comparing this statistic with a table. This question is vital to the validity of the study.

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EJ 057 702 AA 512 410

A STUDY OF ACHIEVEMENT AND ATTITUDE OF SECOND GRADE STUDENTS USING TWO MODES OF INSTRUCTION AND TWO MANIPULATIVE MODELS FOR THE NUMERATION SYSTEM. Knaupp, Jonathan, Illinois School Research, v8 n2 pp27-33, W 72.

Descriptors--*Grade 2, *Academic Achievement, *Manipulative Materials, *Student Attitudes, *Arithmetic, Methods Research

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Larry P. Leutzinger and Marilyn J. Zweng, The University of Iowa

1. Purpose

The stated purpose was "to study the effects of two modes of instruction using two manipulative models for the numeration system on the achievement and attitudes of second grade students." The study later concerned itself "with measuring attitudes of the classes toward learning activities after it was established that achievement gains were made by all four classes."

2. Rationale

Studies dealing with methods and materials used in teaching elementary school arithmetic have, for the most part, focused on achievement gains as the criterion for determining the superiority of one technique or set of materials over another. Yet, it is well documented that the attitudes children possess toward arithmetic influences, to some extent, the learning that takes place. This study attempts to investigate whether or not students develop favorable attitudes toward arithmetic when using manipulative materials or when their teacher uses the materials for demonstration purposes.

3. Research Design and Procedure

Four classes of second grade students from Champaign, Illinois public schools participated in the study for four weeks in April and May of 1970. The class sizes were 15, 17, 25, and 28 and the means of the I.Q.'s of the classes were 104, 110, 116, and 117, respectively.

Classes 1 and 2 were similar in characteristics (size, I.Q., background, etc.) as were classes 3 and 4. Class 1 used a manipulative model made of blocks of wood similar to one recommended by Dienes in an activity-oriented setting. Class 2 used the same model, but the teacher manipulated the materials. Class 3 used ice cream sticks in an activity-oriented setting, and in class 4 only the teacher manipulated the ice cream sticks.

Three exams constructed by the experimeter, together with 19 items from an exam by Ashlock and Welch,[1] were used to test student achievement. Two of the experimeter's tests were used to determine competency in using the algorithms investigated in the classes. The third test of the experimeter involved word problems. The items taken from the Ashlock-Welch exam were used as a measure of the understanding of the numeration system.

The experimeter developed four tests dealing with aspects of student attitudes toward learning arithmetic. Two of these tests were general in nature, and the other two tests were specifically intended for use with the materials and procedures under study. The general tests were used as pretests and posttests while the more specific tests were used only as posttests. Using these tests, the experimeter investigated two attitude variables. The first of the variables was the student' preference for working with manipulative materials of the study. The second variable involved the student's preference for working independently. Comparison between pretest and posttest scores were made for attitude and independence. Class 1 was compared with class 2 and class 3 was compared with class 4 using t-tests to determine if there was any change in attitudes or independence involving the specific activities and procedures of the study.

4. <u>Findings</u>

a. On the achievement protion of the study, there were significant gains by all four classes.

b. Changes in class attitude toward arithmetic as evidenced by the scores on the two general attitude tests were nonsignificant for all four classes and for the entire group.

c. Only one of the classes (#1) tended to become more independent (.20 level of significance) in doing arithmetic.

d. There were no significant differences between the classes on the measures of attitude toward specific activities and procedures used in the study.

e. There is a significant difference (.05 level of significance) between class 1 and class 2 with reference to working independently within the framework of the study.

5. Interpretations

Neither student attitudes toward learning arithmetic nor student independence was changed by using lessons based on manipulation of physical models. The experimeter concluded that age, sex, or intelligence (I.Q.) had no significant bearing on student preferences for different activities.

Abstractor's Notes

The experimeter has gone to great lengths in preparing this study. He has found it necessary to prepare his own tests for both achievement and for attitude. One design for testing attitudes deserves consideration since it appears to be easy to administer, understandable, and reliable. The test uses 5" x 5" sketches of classroom situations involving mathematics. The respondents are asked to describe how they would feel if they were in the sketch by marking one of five faces that show degrees of sadness and happiness.

Although this study contains valuable material for use in future studies, several questions need to be raised. In the first place, although the study deals with achievement, no data are present to support the conclusions made by the experimeter. Is this lack of data significant? Also, the Ashlock-Welch test, which was used as a source for questions to measure understanding, contains items of the following type:

1) Choose the numeral which shows the number nearest to 44 when you count by ones. 39 50 41 49

2) In the row of numerals after the "A" put an "X" on the numeral in which the "4" means 4 hundreds.
   A 402 43 4803 48

These questions, while they do test understanding to a certain extent, do not require experience in the manipulation of materials in order to correctly answer them. There are no questions on the Ashlock-Welch test which ask the student to express the number for groups of tens and ones in numeral form or which require concepts of regrouping to answer. Questions of this type would be more likely to discriminate between the methods of instruction used in the study than the questions which were used.

Another question arises concerning the scores reported on the attitude tests. Are these scores high or low, or are they average? If the scores are high on the pretests, little gain could be expected on the posttests. A group of able second-graders might be at a stage where it is easy for them to abstract and, hence, do well on achievement tests of the Ashlock-Welch type, but they might become impatient when they are asked to manipulate materials for a longer time than they feel is necessary. This resulting impatience might have a negative effect on the attitude of the students.

One remark made by the experimeter is of special interest to the abstractor. It states, "a certain amount of self-reliance is developed by the slower students since they can solve difficult addition and subtraction with no assistance." This seems an important result which should be investigated further.

Larry P. Leutzinger
Marilyn J. Zweng
The University of Iowa

Reference

Ashlock, Robert B. and Welch, Ronald C. A Test of Understanding of Selected Properties of a Number System: Primary Form (Bulletin of the School of Education, Indiana University, Volume 42, Number 2, March 1966).

EJ 059 322 310 SE 506 074

THE LEARNING OF THE VECTOR SPACE STRUCTURE BY SIXTH GRADE STUDENTS.

Lamon, William E.; Huber, Leslie E., Educational Studies in Mathematics, v4 n2, pp166-181, Dec 71
Descriptors--*Elementary School Mathematics, *Instruction, *Learning, *Research, Algebra, Grade 7, Manipulative Materials, Mathematics Education, [*Vector Spaces, Structure (Mathematics)]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Thomas J. Cooney, The University of Georgia

1. Purpose

The authors' main interests were centered in two hypotheses:

"(1) If the rules which define the structure of a game generated by concrete materials, are those which are the axioms of a finite vector space, then sixth grade students can behavioristically demonstrate successful learning when required to identify whether or not a structure is that of a finite vector space.

"(2) Operational achievement will correlate positively with: (a) measures of verbal and/or nonverbal intelligence; (b) scores on each of the Comprehensive Tests of Basic Skills; and (c) chronological age."

2. Rationale

This study is an extension of previous studies by Dienes and Jeeves and Lamon in their investigations of children's ability to learn mathematical structures. Studies by these investigators indicated that children can learn the underlying structures of mathematical groups when exposed to concrete mathematical experiences. Furthermore, the investigations provided evidence that learning was on a structural basis rather than on a stimulus-response basis. Because of these findings and the applicability of Piaget's theory of Cognitive Development, the present authors investigated the learning of a vector space structure taught through the use of games involving manipulation of concrete objects.

3. Research Design and Procedure

The subjects consisted of 32 sixth grade students. To help ensure subjects would have a wide range of ability, selection was based on performance on the Lorge-Thorndike Intelligence Tests.

The instructional program consisted of students working at their own pace using task cards. The cards systematically presented in a game format the four aspects of a vector space, viz., "addition" of vectors to vectors, "addition" of scalars to scalars, "multiplication" of vectors by scalars and "multiplication" of scalars by scalars. These four aspects were referred to as "four levels of mathematical thought." The students worked with the cards for 45 minute daily periods over a time span of six weeks.

To assess performance, both an abstraction test and a transfer test were administered to students completing the four aspects of a vector space. The transfer test involved the presentation of a second finite vector space to students and asked them to complete tables for the four binary operations and compare and contrast the second space with the acquired one. The abstraction test required recognition of a vector space among a series of relevant and irrelevant concrete representations.

Product - moment correlations were calculated to assess the relationship between performance and measures of intelligence. Analysis of variance for the regression of verbal I.Q. (and nonverbal I.Q.) on performance level was applied to test the linearity of the relationship between performance and verbal intelligence (and nonverbal intelligence). "Nonparametric rank correlation coefficients were calculated to assess the relationship between levels and the raw scores on each of the Comprehensive Tests of Basic Skills and between levels of performance and chronological age.

4. Findings

1. Thirteen of the 32 students completed successfully the four levels of mathematical operational thought and met criterion on the transfer task. Two of the thirteen students failed to meet criterion on the abstraction test.

2. "The mean verbal I.Q. score, nonverbal I.Q. score, and chronological age for the students achieving each level gradually increased from level one to level four."

3. A product - moment correlation of .54 ($p < .01$) was obtained for performance level and nonverbal I.Q.

4. A significant F-value ($p < .01$) was obtained from the analysis of variance for the regression of verbal I.Q. on performance level. The F-value for the departure of the mean verbal I.Q. score from a straight line was not significant. The F-value for the mean verbal I.Q. scores of the levels differed significantly ($p < .05$) among themselves.

5. The analysis of variance for the regression of nonverbal I.Q. on performance level yielded the following results. The linear correlation between nonverbal intelligence and performance was significant at the .01 level. The departure of the mean nonverbal I.Q. score for each level from a straight line was not significant. The mean nonverbal I.Q. scores of the levels did not differ significantly among themselves.

6. The nonparametric rank correlation coefficients between levels of performance and reading vocabulary and comprehension were both significant ($p < .001$); between performance and language mechanics, expression and spelling were all significant ($p < .01$); between performance and arithmetic computation, concepts and applications were all significant ($p < .01$). The correlation between chronological age and performance was -.01.

5. Interpretations

1. The findings stated in 1 and 2 above were identified as support for the two basic hypotheses.

2. Based on the findings cited in 6 above, the hypothesis predicting positive correlation between performance and chronological age was rejected. It was also suggested that the understanding and utilization of language was directly related to performance. Furthermore, it was suggested that the understanding of arithmetic concepts and skill acquisition was directly related to the ability to understand the structured basis of a vector space.

3. It was noted that students were highly motivated to manipulate the structural embodiment.

Abstractor's Notes

The authors have produced a study which again provides supporting evidence that some children can learn underlying mathematical structures. This and other studies relating to this issue serve the dual purposes of furthering information about the cognitive development of children and in providing information pertaining to what mathematics can be taught with some degree of success in the elementary school.

There are limitations, some noted by the authors, and concerns related to the study.

1. What was the basis of selection for the subjects, i.e., what criteria were established for acceptance or rejection of subjects? Since the sample was small the selection process is very critical.

2. Hypothesis 1 states, in essence, that sixth grade students can successfully learn whether or not a given structure constitutes a finite vector space. If by "sixth grade students", the authors are claiming existence then their evidence supports the hypothesis. If, however, the hypothesis refers to most sixth grade students, then they have not found evidence to support the hypothesis in question.

3. What was the absentee rate during the instructional program? What feedback and reinforcement, if any, did the students receive?

4. The authors speak of levels of mathematical thought but it is not clear whether they are referring to the learning of the four parallel aspects of the vector space or whether level refers to some hierarchal arrangement alá Bloom or some other hierarchal arrangement.

5. The delineation of the transfer and abstraction tests leaves doubt as to the dichotomy of these two tests and whether they do in fact measure different aspects of mathematical thought. Also the criterion for successful performance was not clearly explicated by the authors.

6. Finally, one can not determine whether students were motivated because of the mathematical experience or because of the opportunity to participate in an experiment.

Apart from these issues, the authors have an interesting study. Replications are needed to further support and substantiate their findings.

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ED 062 138 SE 013 559
THE EFFECT OF CLASS SIZE ON THE LEARNING OF MATHEMATICS: A PARAMETRIC STUDY. Moody, William B. And Others. Pub Date 1972. Note-13p; Paper presented at the meeting of the American Educational Research Association, April 1972.
EDRS Price MF-$0.65 HC-$3.29
Descriptors--*Class Size, *Elementary School Mathematics, Grade 4, *Individualized Instruction, *Research
Expanded Abstract and Analysis Prepared Especially for I.M.E. by M. Vere DeVault, The University of Wisconsin, Madison.

1. Purpose

The major purpose of the study was to determine if small class sizes other than the tutorial setting could result in increased learning.

2. Rationale

Previous research on the relationship between class size and student achievement has provided overwhelming evidence for the conclusion that the number of students within a given class has little influence on learning. The authors consider two major limitations of previous studies. The first is the time existing between instruction and testing. The second is the minimal variability among class sizes under study. The authors set out to minimize these deficiencies by testing immediately following instruction of class sizes of 1, 2, 5 and 23 Ss.

3. Research Design and Procedure

The Ss were 249 4th grade students drawn from three elementary public schools. Eighty-three Ss from each school were randomly assigned to four treatment groups identified as 1-1, 1-2, 1-5, and 1-23. Within each school Group 1-1 consisted of 20 instructional trials in which one teacher instructed one S. Group 1-2 consisted of 10 instructional trials in which one teacher instructed two Ss simultaneously. Group 1-5 consisted of 4 trials with one teacher instructing five Ss and Group 1-23 consisted of one instructional trial in which one teacher taught 23 Ss in a classroom setting.

Teachers for the study were 17 undergraduate junior and senior level elementary education majors who volunteered to participate. Because of the schedules of volunteers, teachers were not randomly assigned to schools. Teachers were randomly assigned, however, within each school. All teachers were assigned to at least two Group 1-1 instructional trial, to at least one Group 1-2 trial, and no teacher was assigned more than one Group 1-5 trials nor to both a Group 1-5 and a Group 1-23 trial. Within those limitations and within space limitations teachers were in other instances randomly assigned. One week prior to the beginning of the experiment each teacher was given a list of 10 specific instructional objectives accompanied by examples and brief mathematical discussions of each. No instructional methods nor techniques were suggested.

All 4th grade students were pretested one day prior to instruction. To insure that Ss were unfamiliar with the experimental contant, only Ss attaining a score of five or less on the 20-item pretest were selected. Less than 1% of the population was thus eliminated.

For instruction the Ss were taken from their regular classroom and placed in the charge of their assigned teacher. Each S was then instructed for exactly 30 minutes. Immediately following instruction all Ss were retested by an experimenter in another area but outside their regular classroom.

Instructional content was directed toward 10 objectives. Each objective was measured by two items on the 20-item post-test.

Illustrative objective:

Rename the product of two different factors with similar exponents as the product of the factors with the common exponent.

Example: $6^2 \times 7^2 = 42^2$

The two items used to measure the objective:

1) $6^1 \times 5^1 =$ ______
2) $2^5 \times 3^5 =$ ______

Split-half reliability for the 20-item test using the Spearman-Brown formula was 0.89. The posttest correlated 0.55 with PMA IQ scores.

4. Findings

The results indicate that class sizes studied strongly affected over-all achievement of the 10 mathematical objectives. Differences among schools were significant at the .01 level and those between class sizes were significant at the .001 level. The means and standard deviations for the posttest scores are reported here.

Group Size		School #1	School #2	School #3	Total
1	$\bar{X}$	10.25	12.30	12.20	11.58
	S.D.	5.84	4.60	4.47	5.01
2	$\bar{X}$	9.65	10.40	9.95	10.00
	S.D.	4.37	3.86	3.50	3.75
5	$\bar{X}$	8.10	7.35	11.85	9.10
	S.D.	4.06	4.84	4.19	4.56
23	$\bar{X}$	5.26	8.83	9.22	7.75
	S.D.	3.70	4.19	3.69	4.21

5. Interpretations

Given the limitations of the present study, the finding that learning varies with individualization of instruction has both practical and theoretical significance. An empirical rationale is supplied for small group remedial instruction in those cases in which additional personnel are available to supplement the instruction of the classroom teacher. The data clearly indicate that for those subjects small group instruction was incremental when compared to large group instruction and that large group instruction is much more efficient in terms of total learning produced. It is tempting to suggest that personnel such as teacher aides might be efficaciously employed to instruct small groups of academically needy students.

Abstractor's Notes

There has long been a need for studies which were designed in a manner to facilitate our analysis of the problems relating to class size. Research to date has failed to provide evidence that classes of 25 learn more than those of 35. The present study takes the case to extremes and identifies that for these subjects there can be little doubt but that differential learning did occur. With replications of the present study, implications move in two directions. On the one hand, the results speak to the increasing interest in providing for individualization of instruction in classrooms. What roles do staff play in individualized contexts? On the other hand, the present study leaves much uncovered ground between its design and that of studies we have come to expect on this topic. Hopefully, researchers will seek answers to many questions which lie between.

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EJ 075 357 450 SE 508 174
LOGICAL THINKING IN ADOLESCENTS. O'Brien, Thomas C., Educational Studies in Mathematics, v4 n4, pp. 401-428, Dec. 1972.
Descriptors--*Deductive Methods, *Logic, *Mathematical Logic, *Research, *Secondary School Mathematics, Learning, Mathematics Education.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by F. Joe Crosswhite, The Ohio State University.

1. Purpose

The purpose of this research was to examine high school student's performance in dealing with the logical operation of implication as measured by items of the following forms:

1. MODUS PONENS
   If the car is shiny, it is fast.
   The car is shiny.
   Is the car fast?
   (a) Yes
   (b) No
   (c) Not enough clues.

2. CONTRAPOSITIVE
   If the car is shiny, it is fast.
   The car is not fast.
   Is the car shiny?
   (a) Yes
   (b) No
   (c) Not enough clues.

3. INVERSE
   If the car is shiny, it is fast.
   The car is not shiny.
   Is the car fast?
   (a) Yes
   (b) No
   (c) Not enough clues.

4. CONVERSE
   If the car is shiny, it is fast.
   The car is fast.
   Is the car shiny?
   (a) Yes
   (b) No
   (c) Not enough clues.

2. Rationale

Previous research, by the author and others, has revealed that children as young as age 6 seem to have little difficulty with inference patterns 1 and 2. However, subjects of high school and even college age answer items of forms 3 and 4 in a consistent and erroneous way answering "No" and "Yes" respectively rather than correctly responding "Not enough clues". The author has termed this behavior "Child's Logic" in contrast to "Math Logic" characterized by correct responses to items of forms 3 and 4.

The rationale for the present study was that it could provide substantiation of the use of "Child's Logic" in yet another population. The study also extends previous research in that it provides a context for the systematic examination of performance on all four inference patterns and the effects of mode (one of four modes, depending on whether the first sentence of an item was $P \to Q$, or $\overline{P} \to Q$, $P \to \overline{Q}$, or $\overline{P} \to \overline{Q}$).

3. <u>Research Design and Procedure</u>

To assess generalizability of performance across context, four tests consisting respectively of items which were causal (Test I), class inclusion (Test II), nonsense (Test III), or random (Test IV) in context were used. Three basic sentences were cast in four modes for each of the four inference patterns yielding 48 items for each test. All items were presented in cartoon format.

The study population consisted of a random half of the students in a private (parochial) girl's high school. Half of each grade level was randomly divided and each resulting group took one of the tests; Tests II and IV being used in grades 9 and 11 and Tests I and III in grades 10 and 12. The total number of subjects involved was 156.

Data analysis consisted of eyeball comparison of the per cent of correct responses for grade, form, mode, form x grade, mode x grade, context x grade, form x mode, form x context, mode x context, form x mode x context, form x mode x grade, form x context x grade, context x mode x grade, and form x mode x context x grade. Data were pooled across grade, form, mode, or context as convenient for these comparisons. No statistical tests were employed and no reliability estimates were given for the tests used.

4. <u>Findings</u>

The number of potential comparisons imbedded in the graphical displays of data which occupy the major portion (pp. 406-424) of space devoted to this research report exceeds the number of subjects involved in the study. While the author does not discuss all these comparisons, only the grossest sort of summary of findings can be reported in this brief abstract.

The consistent use of "Child's Logic" in dealing with open (INVERSE and CONVERSE) items was reaffirmed in this population. Unexpectedly, the use of "Math Logic" on inverse items was double that on converse items. Modus Ponens was, as expected, an easy inference pattern for these subjects with the contrapositive form consistently more difficult (a spread of 32 per cent points in the overall data). No dramatic gains in subject performance over grade were observed. Context effects were noted throughout the data but were not consistent accross forms. Mixed effects were also noted in the mode data.

5. Interpretations

The previously reported "Child's Logic" would suggest that subjects interpret $P \rightarrow Q$ as $(P \cdot Q) \vee (\overline{P} \cdot \overline{Q})$ instead of $(P \cdot Q) \vee (\overline{P} \cdot \overline{Q}) \vee (\overline{P} \cdot Q)$. This earlier interpretation does not adequately explain the data obtained in this study wherein a widespread inability to recognize the validity of contrapositive inference and a converse-inverse imbalance were observed. The author conjectures the existence of a different "Child's Logic", a semi- "Child's Logic", in which $P \rightarrow Q$ is interpreted as "P yields Q and Q yields P and nothing else". He indicates that the existence of two "Child's Logics" explains more adequately the subjects' performance on both open and closed implication items than has been the case in his earlier research.

Abstractor's Notes

The overall conceptualization of this study is to be commended. The tests employed seem to have been well-designed to yield interpretable data relative to an interesting and important question. It is unfortunate that limitations in the research design and, particularly, in the way the data were handled might obscure important results even for the non-casual reader.

Although no statistical tests of significance were employed, the language used in drawing comparisons is suggestive. Moreover, it is suggestive of longitudinal comparisons although the study is clearly cross-sectional. For example, in drawing grade-level comparisons the following descriptors are employed for the differences indicated: little gain (4.91), very small gain (2.81), relatively constant (2.28), dropped slightly (1.82), small rise (0.68), rose sharply (10.65). At another point, in comparing scores for the total population, the author uses "slightly exceeded" for a difference of 0.48 and "virtually no difference" for a difference of 0.88. When one considers that these comparisons involve a wide variety of data pooling under a design in which each subject took a single test, the comparisons become even more difficult to interpret. The conversion from item scores to percentage scores magnifies the differences observed but hardly clarifies them.

One would hope this research could be replicated under a design that permitted a more classical analysis of the data.

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ED 062 137 SE 013 557
AN INVESTIGATION IN THE LEARNING OF RELATIONAL PROPERTIES BY KINDERGARTEN CHILDREN. Steffe, Leslie P., and Carey, Russell L., Pub. Date, 1972, Note--31p.; Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, Illinois, 1972.
EDRS Price MF-$0.65 HC-$3.29
Descriptors -- *Concept Formation, *Conservation (Concept), *Elementary School Mathematics, *Instruction, Kindergarten Children, Learning, Logic, Relationship, *Research

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Elizabeth H. Fennema, The University of Wisconsin, Madison

1. Purpose

The major purposes of this study were to investigate (1) improvement in the usage of properties of equivalence and order relations by five-year-old children under specified instructional conditions and (2) to explore the interrelationship of time of acquisition of equivalence and order relations when represented by matching (one-one correspondence) or length.

2. Rationale

Although intervention training has been shown not to be effective in hastening the development of logical thinking capabilities, previous studies have dealt mainly with the idea of conservation. This study investigated the possibility of increasing the ability to use relational ideas, which are built on the idea of conservation, by specific instructional intervention. Piagetian oriented researchers have found that acquisition of length follows acquisition of conservation by matching. This study explored the question of an analagous time lag in the development of usage of the relational properties of equivalence and order when represented by discrete objects (matching) or by length.

3. Research Design

48 kindergarten children, divided into groups of 6, were given a pre-treatment composed of 14 instructional sessions which were designed to define operationally by length and matching, the properties of equivalence (as many as, as long as) and order (more than, fewer than; longer than, shorter than). Three dimensional items plus verbal comments by the instructor were used exclusively in teaching. Following the pre-treatment, an individual pretest consisting of subtests which measured conservation, equivalence and order was given and on the basis of performance on this test subjects were assigned by matched pairs to one of two treatments.

In the Classification Treatment objects or lengths were considered as representative of another set of objects or lengths and in the Standard Treatment no attempt was made to consider objects or lengths as representative. In both treatments programmed questioning and discussion techniques were utilized which involved children in work both with sets of objects and linear objects. Both treatments were composed of 12 instructional sessions. Following the experimental treatments, subjects were again tested individually on a posttest consisting of the same items as the pretest plus additional items measuring transitivity and the asymmetric property. Criteria were established and subjects were rated on a 0,1 basis indicating that they either did or did not perform at criterion level on the subtests of both pre and posttests. "0", "+", or "-" were assigned indicating change (or its lack) in score from pre to posttest. Matched pairs on the two treatments were assigned an ordered pair indicating each individual's rating on a 0,1 basis. The sign test was used to test the relative effectiveness of the two treatments and the McNemar test for significance of change was used to test the significance of the gains for the entire group from pre to posttest. Contingency tables were used to analyze the order of development of the various properties exemplified by length or matching.

4. Results

There was no evidence found on either the pre- or post-test that supported the belief that (1) ability to conserve matching relations precedes ability to conserve length relations: or (2) knowledge of order represented by matching relations precedes knowledge of order represented by length. There was some evidence to support the idea that ability to use the asymmetric property represented by sets. There were no significant differences evidenced between treatments. When the results from the two treatments were combined, a significant change in usage of transitivity, conservation of length and matching relations were found. It was concluded also that if a child could conserve relations on the pretest it would seem that he could learn transitivity more easily than those children who could not conserve on the pretest. The authors also conclude that once improvement was shown in the usage of the properties studied that "under controlled instructional conditions, the logical thinking of children as it pertains to properties of matching and length relations can be improved long before second grade."

Abstractor's Notes

Several questions were not answered in the report.

A. Is it necessary to write research reports in a form such as this one was written? It is accurate reporting but the audience is limited by the terminology and structure of the language.

B. The formal hypotheses (p. 4) do not reflect the specific purposes (p. 1) of the study.

C. Who served as instructor(s) in both the pre- and experimental treatments? The teacher variable is perhaps one of the most important ones in an experimental study of this type and yet how it was controlled is not mentioned.

D. Is the statistical treatment adequate? In only one instance is inferential statistics used. The size of the N would permit more sophisticated analysis to be done.

E. What are some of the data? Tables are not labelled well (i.e. Matching by Length Relations could mean at least two things. Matching X Length would be better.) It is extremely difficult if not impossible to match the written discussion with the data presented in tables.

F. Data on retention of the usage of order and equivalence properties would have been helpful. The major conclusion of the study concerning improvement of logical thinking of young children might not have been valid if such retention data had been obtained after an appropriate interval of time. Other studies have shown that maintenance of change in logical thinking patterns is much more difficult than effecting the change originally.

G. What are the implications of this research for the development of early education mathematics curricula?

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ED 064 147 24 SE 014 126
A STUDY OF NUMBER CONSERVATION WITH TASKS WHICH VARY IN LENGTH, AREA AND VOLUME. FINAL REPORT. Taranto, Maria and Mermelstein, Egon, Hofstra Univ., Hempstead, N.Y., National Center for Educational Research and Development (DHEW/OE), Washington, D.C., Pub. Date Jun 1972. Note--56p.
EDRS Price MF-$0.65 HC-$3.29

Descriptors--Cognitive Development, *Conservation (Concept), Elementary School Mathematics, *Learning, Mathematical Concepts, *Mathematics Education, Number Concepts, Preschool Learning, *Research

Identifiers--*Piaget (Jean)

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Leslie P. Steffe, University of Georgia

1. Purpose

Taranto and Mermelstein tested the three hypotheses that (1) the order of development of ability to pass number conservation tasks, which differ on the dimensions of length, area, and volume, will parallel the order of development of conservation of length, area, and volume; (2) where the elements of the number conservation task form continuous collections within each dimension (length, area, volume) vs. where the elements of the task form discontinuous collections within each dimension, number conservation will be mastered by a child on the task involving discontinuous collections prior to mastery of number conservation on the task involving continuous collections; and (3) given a number conservation task that involves transfroming discontinuous quantity to continuous quantity, the child must be facile not only with number conservation but also. conservation of amount.

2. Rationale

Utilizing cross-sectional data, it has been found that number conservation emerges after approximately 5 years of age, length and area conservation emerges after approximately 7 years of age, and volume conservation emerges after approximately 12 years of age. Taranto and Mermelstein suggest that the stated order of development of the above conservations may not be identical to the order of conservation within an individual. They suggest that, within an individual, an order of development between length conservation and area conservation may be found because the definition of area "includes" length. Number conservation tasks, if cast in terms of length, area, and volume, may follow an order of difficulty corresponding to the sequence of development of length conservation, area conservation, and volume conservation. Moreover, because conservation of discontinuous quantity precedes conservation of continuous quantity in development, some further clues as to the nature of conservation of number may be obtained by casting number conservation tasks across the two dimensions of transforming discontinuous quantity into discontinuous quantity and transforming discontinuous quantity into continuous quantity.

3. Research Design and Procedure

A pool of 80 children, 40 female, and 40 male, ranging in age from 4 years to 7 years of age were used as subjects. Forty children, 10 four-year-olds, 10 five-year-olds, 10 six-year-olds, and 10 seven-year-olds, were administered eight number conservation tasks (Set I) and 40 different children partitioned analogously by age, were administered eight different number conservation tasks (Set II). Three children of each 40 were dropped from the analysis.

The crucial difference in the two sets of tasks was that in Set I the difference between the two collections in each task was based on size, whereas in Set II, the difference was based on shape. The tasks are described in part in Table 1. In Set I, the size differential was in the ratio 2:1 whereas in Set II, the objects were of the same size.

Table 1
Number Conservation Tasks

	Classic Task: A	Sticks Apart: B	Sticks Together: C	Trees on Field: D
Set I	chips: 8 blue; 8 red	7 long; 7 short	7 long; 7 short	9 tall; 9 short
Set II	chips: 8 blue; 8 red	two sets of 7/set	two sets of 7/set	two sets of 9/set

	Tile Floors: E	Snowball in Jars Task: F	Block House: G	Water Glass Task: H
Set I	9 large; 9 small	9 large; 9 small	8 large; 8 small	7 large: 7 small
Set II	two sets of 9/set	two sets of 9/set	two sets of 8/set	two sets of 7/set

The classic number conservation task (A) was the same for each set of tasks. In each task, an optical one-to-one correspondence was established between the two collections of objects. In Task A, the chips in one of the rows was then extended beyond the length of the chips in the remaining row; in Task B, discontinuous rows of sticks were made; in Task D, the trees were placed on different "fields"; in Task E, tile "floors" were made; in Task F, the snowballs were placed into jars; in Task G, block houses were made; in Task H, the glasses of water were poured into containers. In Set I, the set of larger objects always occupied a larger space than the set of smaller objects in their final states and in Set II, a perceptually conflictive situation was formed by the sets of objects in their final state. A clinical method of interviewing was used where the child and the experimenter were actively involved in manipulation of materials. A second trial was given in some cases for clarification. It involved an exchange of material by the experimenter and child, but was identical in all other respects to the initial trial.

A child was placed into three stages with regard to each task: Stage 3 if he asserted conservation of number (on both trials when two were given) and argued by reversibility, compensation, or identity; Stage 1 if he asserted nonconservation of number (on at least one trial when two were given); and Stage 2 if on either trial 1 or trial 2 he vacillated between nonconservation and conservation of number. Two tasks were administered per sitting, with tasks randomized for each child.

The general hypothesis for the experiment was that $r_a \geq r_b \geq r_c \geq r_d \geq r_e \geq r_f \geq r_g \geq r_h$; where r_i represents the response score on task i. For any child, if it was not the case that $r_i < r_j$ for $i < j$, then the hypothesis would be confirmed for that child. On the basis of random responses, the probability of the hypothesis being confirmed is less than .007. Also, a scalogram analysis was done to determine the scalability of the items.

4. Findings

(1) For all eight tasks of Set I, 13 children manifested confirmation of the general hypothesis and 24 did not. Of the 13 confirming instances, 10 were unitary patterns (all 1's, 2's or 3's). In Set II, 16 children manifested confirmation of the hypothesis while 21 did not. Of the 16 confirming instances, 13 were unitary patterns.

(2) For the discrete tasks of Set I (B, D, and F), 21 children manifested confirmation of a modified general hypothesis and 16 did not. Of the 21 confirming instances, 18 were unitary patterns. For the continuous tasks of Set I (C, E, and G), 24 children manifested confirmation of a modified general hypothesis and 13 did not. Nineteen of the 24 confirming instances were unitary in nature. The patterns were similar for Set II.

(3) For the length tasks (B or C), area tasks (D or E), and volume tasks (F or G) of Set I, 28 children manifested confirmation of a modified general hypothesis of which 25 were unitary in nature. Nine children did not manifest confirmation of the hypothesis. A score on the two tasks for each dimension was the greater of the two scores. The pattern was similar for Set II.

(4) For the discrete and continuous dimension for each of the two length, area, and volume tasks for Set I, 28 children manifested confirmation of a modified general hypothesis while nine did not. Of the 28, 22 were unitary in nature. The pattern for Set II was similar.

(5, Using Green's scalogram analysis, no unique ranking of difficulty was found for the items.

5. Interpretations

Taranto and Mermelstein included the following points in their discussion of the study.

(1) What seems clear from the data . . . is that within any individual . . ., the strict hierarchy interpretation [of the tasks] is confirmed in few instances . . . .

(2) Support for the developmental nature of number conservation is evident. Few nursery school and kindergarten children conserve on all eight tasks while . . . almost all second graders conserve on all the tasks.

(3) Presenting conservation tasks to children in any prescribed logical order, with the belief that they will assimilate them in that order, seems questionable as a means of training or teaching.

(4) Relative to number conservation tasks involving length, area and volume, both the constructed probability distribution analysis and the scalogram analysis suggest that the order of acquisition of these tasks is individual.

Abstractor's Notes

Taranto and Mermelstein base their basic hypothesis of the study on the horizontal differentials observed in conservation experiments--conservation of volume occurs as a formal operational phenomena--and on the fact that length is "included" in the definition of area. Piaget, Inhelder, and Szeminska, in the The Child's Conception of Geometry, make a sharp distinction between volume as "occupied space" and "interior volume." Conservation of volume as "interior volume" occurs at the concrete operational stage but not conservation of volume as "occupied space." In their basic hypothesis, then, Taranto and Mermelstein had to be viewing volume as "occupied space." The way their tasks were constructed harked back to the second chapter of Piaget's The Child's Conception of Number where he used containers of different shapes to study conservation of discrete quantity, thus extending his conservation of water experiments of the first chapter of that book. Taranto and Mermelstein's findings are consistent with the data presented in Chapter II of _The Child's Conception of Number_ and are consistent with the development of conservation of "'interior' _and_ 'occupied' volume" due to the way their tasks were constructed. I hypothesize that if they cast their "conservation of number" tasks in terms of "occupied volume," they would observe a horizontal differential in "conservation of number."

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ED 061 097 SE 013 565
THE CHILD'S INTRODUCTION TO MATHEMATICS: A TRANSFER MODEL BASED IN MEASUREMENT. Van Wagenen, R. Keith, Pub. Date, Apr 1972. Note--20p Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, Illinois, April, 1972.
EDRS Price MF-$0.65 HC-$3.29
Descriptors--Arithmetic, Conservation (Concept), *Elementary School Mathematics, *Instruction, Instructional Aids, Kindergarten Children, Learning, *Measurement, *Research

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Doyal Nelson, University of Alberta, Edmonton

1. Purpose

To determine whether teaching the kindergarten child to use "units of measure" in connection with continuous quantities would provide a transfer base for successive new concepts not presently being provided by the emphasis on counting.

2. Rationale

The authors are interested in finding support for a teaching tactic different from counting for introducing mathematics to young children. They claim that in a program where a child has to count things the child is distracted from the mathematical aspects of the objects to be counted by the nature of the objects themselves. The reason, they claim, is that a child's early experiences are largely qualitative and that the attention he gives to the discrete objects as he counts only encourages him to confuse nominal aspects of the process with the qualitative aspects of the objects. The authors state that a general grasp of "correlational relations of numbers with continuous variables" is what the child needs as a basis for attaining an orderly body of knowledge. They mean, one would presume, that the child should be taught to measure as a basic process.

They argue that by measuring continuous variables on a ratio scale the teacher and learner have before them all the "subject matter" there is to learn.

They call on evidence from Gal'perin and Georgiev to support their contention that much mislearning takes place when the child is required first to count, then to add, then to subtract and so on.

The authors in emphasizing the early use of measuring claim to have attempted to make use of the transfer facilitators listed by Ellis.

1. Practice should be exceptionally thorough in the early stages of skill and concept developments.

2. A variety of examples are important to the comprehension of concepts and principles.

3. General principles must be understood before significant transfer takes place.

4. Mediation due to a network of associative linkages between tasks will lead to transfer but the necessary preconditions include development of a relevant language (in this case mathematical).

They claim to have attempted to devise an organized sequence of measuring activities which more or less meet the requirements of maximum general transfer in the child.

3. Research Design and Procedure

To treat their hypotheses of the affectiveness of teaching measurement they first selected twelve of the lowest achieving children in a middle-class kindergarten (average age about 5-1/2 years).

They used a pretest-posttest-control group design. The twelve children were given a mathematics ability test (a 61 item test devised by the authors) and a conservation ability test (the 24 item test devised by Goldschmidt and Bentler) at the beginning of the experiment. Then the children were ranked and matched in pairs according to their mathematics ability scores. The matched pairs were randomly assigned to control and treatment groups.

The experimental group was then given a 17 day instructional program in "measurement" while the control group presumably received regular instruction in mathematics during that period. Five days after the instruction, both groups were given the mathematics ability and the conservation tests used in the pre-test.

Although there is no mention of the kind of instruction the control group was receiving, there is a clear description of the "measurement" program provided for the treatment group.

"Unit of measure" was developed by having the child involved in measuring such objects as a table with various objects such as a book. The idea of using one object repeatedly to obtain and measure was first established through a variety of tasks. Then the child was taught to use multiple units to obtain a measure. All measures were initially devised to come out "even".

Once the child had obtained some skill in these basic processes his measuring was done with prepared dowels. For example, if he measured a table with a book he would be required to find a dowel as long as the book and make the measurement again.

The child was then moved to a rather complicated electro-mechanical device on which he could do various measuring tasks. Essentially the device was composed of a tray on which an object to be measured could be laid and dowels placed over it so that a count could be made of the dowels. There was also a moving slide attached which could be moved to the extremity of the object to be measured. Various counters and dials for adding and subtracting measures were incorporated into the device. The instructional program involved a variety of activities on the electro-mechanical device and provided for a variety of practice. The children were taught individually and the average time spent with each was about 4 hours.

4. Findings

The means of the mathematics and conservation test scores are shown in the following table.

Means of Mathematics Ability and Conservation Ability
Pre-and Post Test Scores

Group	Pretest		Post-test	
	Mathematics	Conservation	Mathematics	Conservation
Treatment	32.67	3.17	37.66	6.50
Comparison	33.33	7.83	36.50	6.17

The authors claim to have used a technique which would attribute .46 of the total variance to result from treatment effects (a relatively high value, they say) and .37 of the total variance in the conservation test to be a result of the treatments.

The authors claim that their study supports that learning hierarchies studied in the laboratory correspond in significant detail to the more complexly associated behaviors in real life. They see cause for including much more measurement in early mathematics learning.

Abstractor's Notes

It would appear that the authors are assuming a significant increase in the mathematics ability and the conservation ability of the six kindergarten children as a result of "measurement" treatment. A difference in growth of two or three points on a 61-item mathematics ability test is not strong evidence on which to base such an assumption. One suspects that the errors in the test alone might very easily be great enough to account for any apparent differences in mathematics ability between the two groups.

The increase in conservation scores in the treatment group could be a result of their being better able to conserve length as a result of treatment experiences in "measuring" lengths and comparing them. It would have been interesting to see on what items of the Goldschmidt-Bentler test these children showed improvement. Anyone who works with kindergarten children knows that they don't normally conserve length that early, but that experience of the kind provided in this study can have a profound influence on ability to conserve length.

I was disappointed that there was no inkling in the report about what happened to the control group in the 17 day period.

Even though the hypothesis tested here gets only meagre statistical support there are some strengths in the study.

Both the sequence and timing of the "measurement" activities the authors have devised are useful information to mathematics educators. They have constructed an intriguing and valuable device around which many such activities can be built. There is a continuing need for many studies of this kind to be conducted using more children with varying instructional schedules.

In the final analysis the child needs a process called counting to deal with discrete objects and a process called measuring to deal with continuous quantities. Both of them are important in his real world. We need effective instructional procedures and sequences to teach both. It is still an open question whether a great advantage can be gained in kindergarten by an initial emphasis on continuous quantities rather than on discrete objects in learning about number.

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